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SOME OBSERVATIONS ON TRANSPORT PROCESSES IN THE WAKE OF A SPHERE IN LOW SPEED FLOW

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Abstract—This paper is concerned with a study of the transfer mechanisms in the wake region of a sphere in low speed flow. The main features of the work are an interpretation of some visual observations of the velocity field in the wake of a sphere and a simple quantitative examination of the heat- and mass-transfer rates which would occur for the particular regime of flow. In the experimental work, attention was focused on the fluid flow immediately adjacent to the surface. As a result of this technique, a simple model for the wake flow near the surface was devised and with this model, a satisfactory correlation of the experimentally measured transfer rates of earlier work is shown to be possible. The working fluid in the experimental study was water and the maximum value of the Reynolds number was 410.

NOMENCLATURE

- D, diffusion coefficient in mass transfer;
- d, diameter of sphere;
- h_d, $(= m/\Delta\rho)$ mass-transfer coefficient;
- m, mass flux;
- u, fluid velocity;
- ν, kinematic viscosity;
- $\Delta\rho$, difference between the concentration of the diffusing substance at the surface and the concentration at the edge of the mass-transfer boundary layer;
- Re, $(= u_\infty d/\nu)$ Reynolds number;
- Re_w, $(= \bar{u}_w d/\nu)$ wake Reynolds number;
- Sh, $(= h_d d/D)$ Sherwood number in mass transfer;
- θ, angle measured from front stagnation point.

Suffices

- 1, at the edge of upstream boundary layer;
- 0, stagnation point;
- ∞, main stream value;
- w, wake value, or pertaining to the edge of the "wake boundary layer";
- s, separation point.

A bar over a symbol means "the average value of".

INTRODUCTION

An understanding of the transfer processes in the wakes of blunt bodies is of major interest to the aerodynamicist and the engineering scientist. It is, however, understandable that less success has been achieved in the solution of the wake problem than in other common flow situations. Attempts to describe a wake flow mathematically results in equations which are extremely complex and their solution is fraught with insurmountable difficulties. Experimentation is also difficult with the result that the interpretation of the observations and results has frequently led to conflicting ideas. The general behaviour of the flow in the wake region of a sphere is, however, fairly well known and might be summarized briefly in the following paragraph.

It is well established that there are essentially two regimes of wake flow. The first regime might be described as steady, being characterized by standing vortex ring of axisymmetric shape at the rear of the sphere. This occurs at relatively low flow rates. Increase in the stream velocity ultimately results in a quasi-steady wake flow of a complex nature: a decrease in the stability of the vortex ring as it grows results in the shedding of the vortex. Soon after another vortex is created, and this finally detaches with the same fate as its predecessor. In this regime of flow, there is. therefore, some periodicity, a phenomenon which can be easily detected by experiment. Other interesting features of the flow are to be observed with further increases in the velocity. Helical movements of vortex filaments in the wake have been observed, the axis of the flow turning about the sphere axis which is parallel to the stream direction. A detailed account of the properties of the wake, and the observations of earlier experiments are given in reference 1. It is noteworthy that there still remains considerable uncertainty as to what velocities (characterized by the Reynolds number) the aforementioned phenomena occur. Clearly, this is a result of experimental difficulties and the simulation of the idealized flow model. Stream conditions vary considerably and it would appear that this is the most important factor. It is almost certain however that the stable vortex ring pattern exists in a turbulence free stream in a range of Reynolds number between about 10 and 150. It is this flow pattern with which we shall concern ourselves in this study.

The present investigation has its origin in a study of the evaporation of small liquid water droplets in a superheated steam atmosphere. Similar problems are to be found in spray drying in the chemical engineering industry, the vaporization of liquid fuel drops in internal combustion engines and meteorological studies. The relative velocities between the drops and the gas or vapour can be sufficiently small in some applications to make the flow Reynolds number within the range to be considered here.

The overall effects of transfer processes involving surface heat or mass fluxes are adequately correlated and many equations exist for the determination of the transfer rates. Reference may be made to any standard heat transfer text for examples of these equations. The majority of these equations are of a semi-empirical nature being determined by the use of the idea of similarity. Fortunately, this method of correlation overcomes the problems of the wake in that the wake effects are included in the overall performance figure for the sphere. The different patterns of flow which occur in both the attached and separated flow regions are allowed for by restricting the use of the equations to particular Reynolds number ranges. In this way, the contribution of the wake is further masked and a detailed study of the wake flow is eliminated.

Theoretical analyses of the transfer processes around the surface of a sphere have been made. The majority of these analyses deal with the boundary-layer flow on the front of the sphere. The attached boundary-layer flow mechanisms may be calculated either by a direct solution of the basic equations or by using the boundary-layer integral method of von Kármán. Typical works employing these methods are those due to Frössling [2] and Brown, Pitts and Leppert [3] respectively. Some theoreticians allow for the wake region by estimating its contribution and then adding it to the boundary-layer effect. The use of the Reynolds analogy has sometimes been employed in studies of momentum and heat transfer. Tang, Duncan and Schweyer [4] for example have employed this idea. There is, however, no real justification for using the analogy as an examination of the fundamental boundary-layer equations for this case will indicate. Because the transfer rates in the wake at small Reynolds numbers are themselves relatively small, an estimate of the wake contribution may be seriously in error without significantly affecting the overall performance. For many engineering purposes, such a procedure may be adequate, but to the enquiring mind some direct study and calculation of the wake effects is imperative. A completely general treatment of the wake is probably an impossible task, but an analysis of a particular regime of the flow is a useful contribution in this direction.

In many of the applications mentioned previously, it is possible that the flow condition is that in which there is a well defined vortex flow in the wake. It is for this particular situation that the present experimental study and analysis has been made.

EXPERIMENTAL APPARATUS AND PROCEDURE

A simple flow visualization technique was devised. The apparatus for the experimental work consisted of a $\frac{7}{8}$ in. diameter steel sphere supported by a single hollow tube through which dye could be fed to a small opening $\frac{1}{64}$ in. diameter on the surface of the sphere. The sphere whose surface was painted grey was

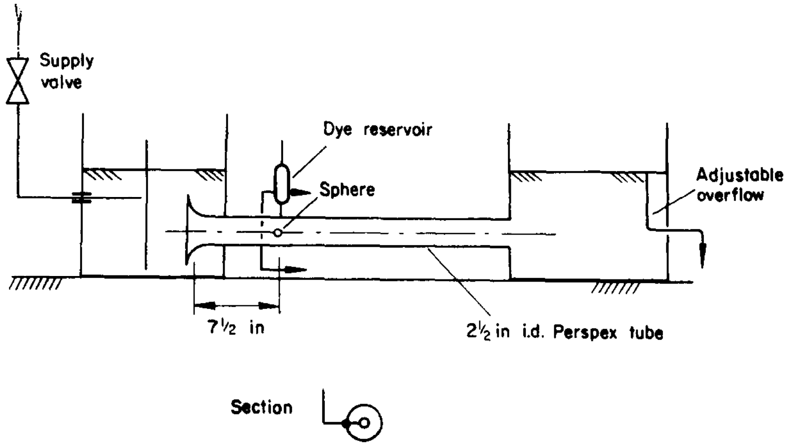


FIG. 1. Experimental apparatus (not to scale).

located in a 2½ in. i.d. Perspex tube close to the entry bell mouth as shown in Fig. 1. The working fluid was water, in which case small stream velocities produced the required Reynolds number range. It was possible to rotate the sphere about an axis perpendicular to the main flow facilitating injection of dye at any angular position. The rate of fluorescence dye injection could be carefully controlled by vertical adjustment of the dye reservoir. For a given Reynolds number and angular position of the dye injection point, the reservoir was first positioned such that the dye was contained in the passage just below the surface of the sphere. The main purpose of the experiment was to study the flow very close to the wall, although observation far from the wall were also desirable. The techniques involved are as follows:

(1) To release dye into the fluid immediately adjacent to the surface, the reservoir was raised slightly to overcome the pressure and surface tension effects at the dye outlet.
(2) By further increasing the height of the reservoir, dye could be introduced into wake regions far from the surface of the sphere.

The main stream flow rate was controlled by adjustment of the water supply valve and the overflow at outlet. Disturbances in the flow were reduced by baffling the flow prior to entry to a carefully faired inlet bell mouth. A considerable length of pipe downstream of the sphere was provided to eliminate exit effect. The flow rate, and hence the Reynolds number was determined by measuring the discharge in a graduated flask. Before making observations, a sufficiently long time elapsed and external disturbances were avoided. The flow pattern was recorded photographically with carefully timed exposures using a green filter and extension rings for close-up photography, average fluid velocities being determined by the distance-time technique.

EXPERIMENTAL RESULTS AND OBSERVATIONS

The existence of a stable vortex ring of axisymmetric shape for low Reynolds numbers in a turbulence free stream is shown in Figs 2(a) and 2(b). This observation was made by photographing the history of dye which was carefully injected at the rear stagnation point of the sphere in the manner described in the previous section under (1). The pattern of the flow under such conditions is typical of the observations of earlier experimenters. A steady reversed flow from the rear stagnation point to the separation point is an important characteristic. Fluid flows from the wake region at a velocity very much smaller than the free stream velocity along the axis of the sphere towards the rear stagnation point. This fluid has a larger potential (i.e. temperature or mass concentration) than the main stream. Another important feature is the stationary eye of the vortex. Little dye reaches this region suggesting that in a heat- or mass-transfer process, the potential there is small. The effect of stream turbulence is characterized by an unstability of the wake. Figure 3 shows one of a series of photographs taken when turbulence was induced in the stream upstream. This pattern of the flow should be compared with that shown in Fig. 2(b) which illustrates the stable situation at the same Reynolds number.

Higher Reynolds numbers tests were conducted as a matter of interest, but because we are restricting our study to the low velocity range, these results are not included here.

In transfer processes in boundary layer and duct flows, the gradients of potential are most marked in regions immediately adjacent the flow boundary. It is well known that the greatest changes in velocity, temperature and mass concentration occur in relatively narrow regions near the surface. The major resistance to the flow of heat in a fluid, for example, is concentrated in fluid layers close to the wall, and because of this attention must be focused on that region. We have noted that little has been done in this direction in wake flows near the surface of a sphere. The conventional methods of measuring velocity using Pitot tubes or hot wire anemometry present practical difficulties. The velocities encountered are very small and the presence of instruments in the wake produce disturbances which cannot be tolerated. The injection of dye as employed in the main flow experiments, however, affords a reliable method of estimating the flow field very near the surface. In our work, we have developed the dye injection technique to produce a reliable means of determining the velocity field near the surface. The velocities in the main wake flow have already been studied by Nisi and Porter [5]. These workers photographed smoke particle streaks to determine the transverse and axial velocity components. We have made observations close to the wall. Figure 4 shows the behaviour of a small quantity of dye injected at the rear stagnation point into the stream. The preferential direction taken by the dye when injected at stagnation point is of little consequence. In this test, which was conducted at $Re = 72$, the injection point is to the left at the lower edge of the picture. Very close to the wall there appears to be no dye. The circumferential velocities of the particles of dye increase with increasing wall distance up to a point where the wall distance is about 5 per cent of the diameter of the sphere. The apparent absence of dye is a result of the steep velocity gradient adjacent the wall. Figure 4 shows that the maximum circumferential velocity can be observed by releasing an extremely small quantity of dye. It should be realized that the shape of the dye-line is not the velocity profile at the instant at which the photograph was taken. The time taken for the dye front to traverse the rear surface up to the point of separation was measured. Some difficulty was encountered in deciding the starting and finishing times for the flow from the rear stagnation point to the point of separation, but good reproducibility of the measurements enables us to put confidence in our results. Our findings are presented in Figs. 5 and 6 where the so called "average velocity at the edge of the wake boundary layer" is plotted against the main stream velocity. Despite the uncertainty concerning the range of Reynolds number in which the flow is of the

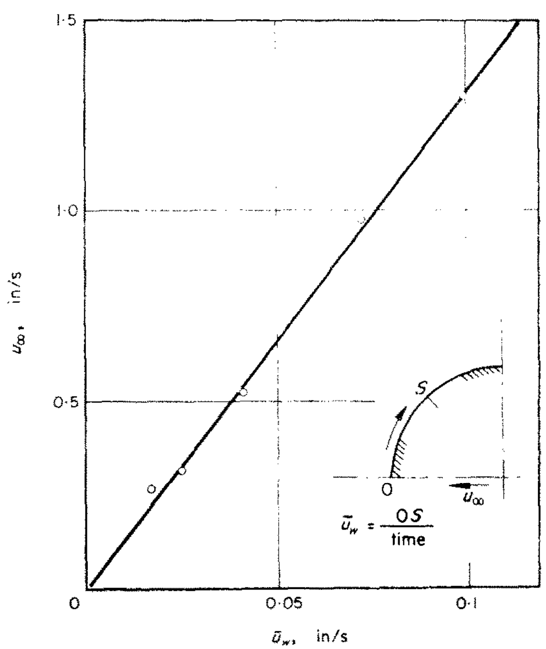
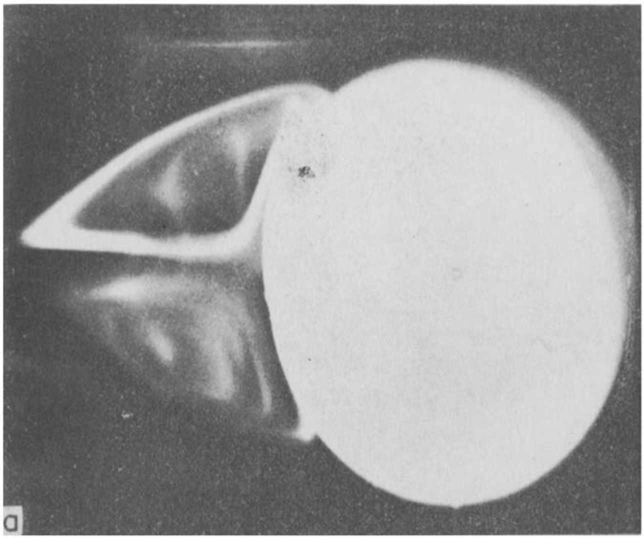
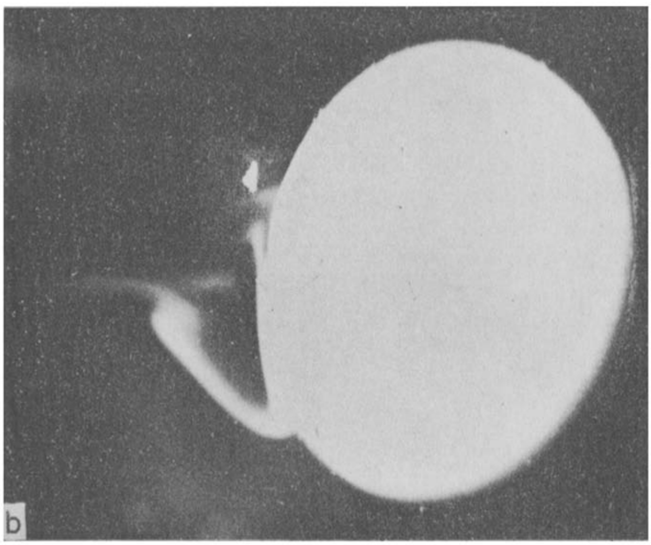


FIG. 5. Relation between wake velocity and mainstream velocity.



Re = 37



Re = 65

FIG. 2. The wake behind a sphere in low speed flow.

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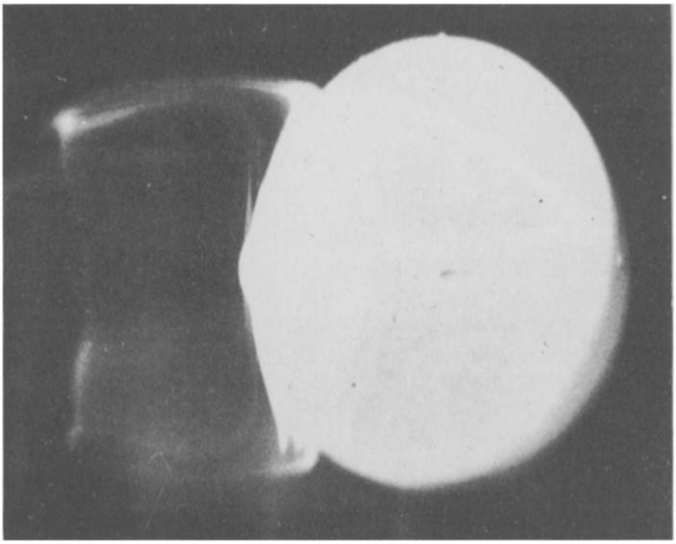
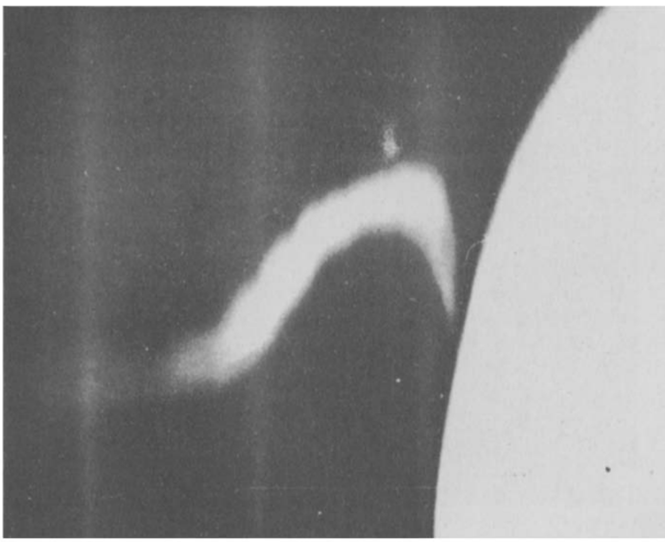


$Re = 65$

FIG. 3. Unstable wake in a turbulent stream.



$Re = 72$

FIG. 4. Flow in the wake near the surface of the sphere.

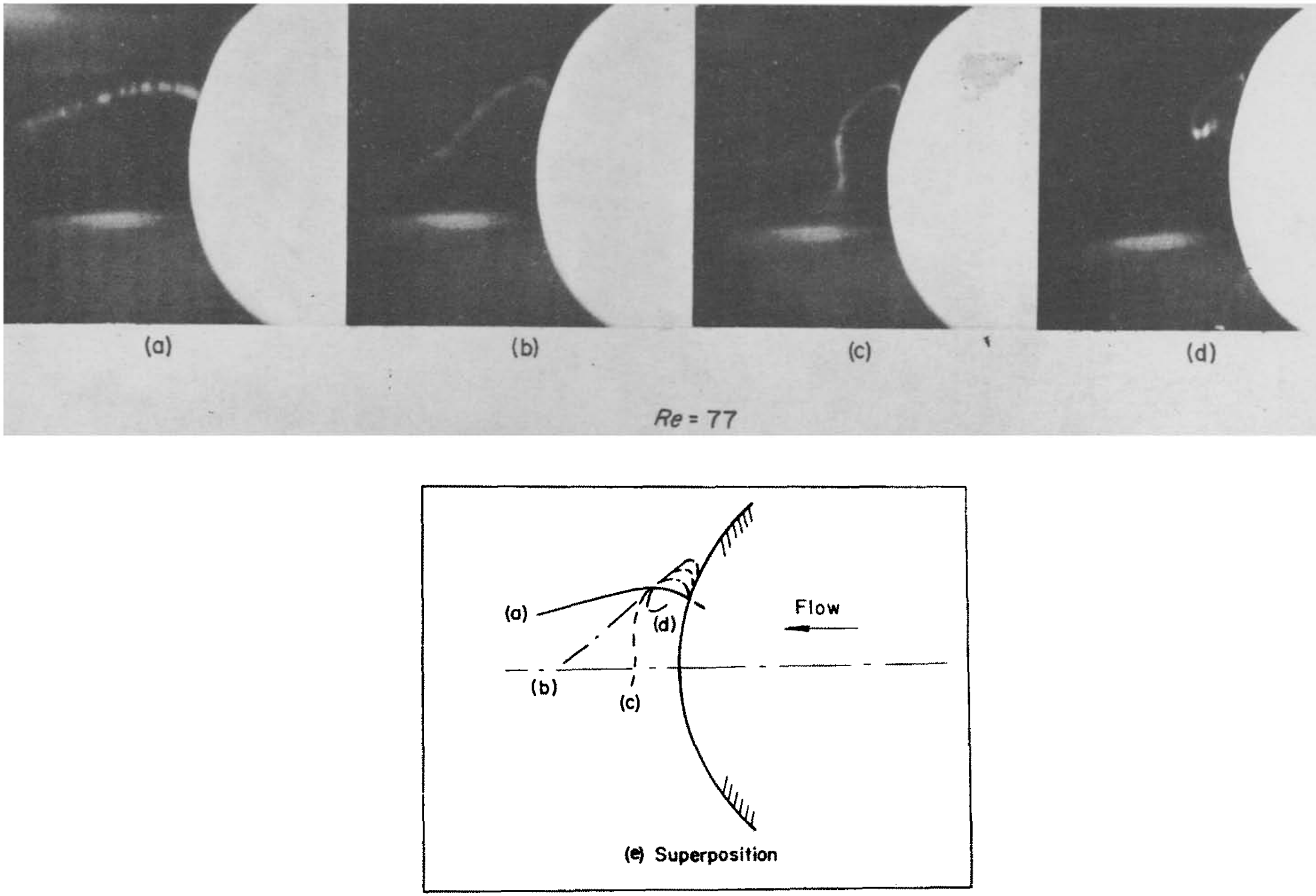


FIG. 7. The nature of flow in the wake region.

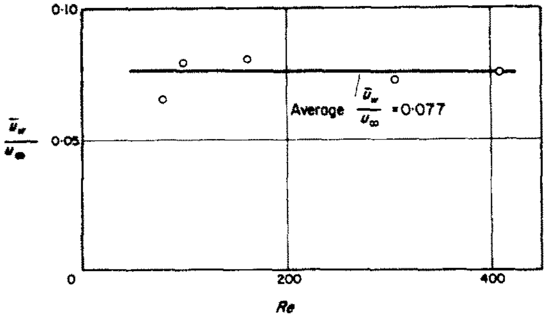


FIG. 6. The wake velocity–mainstream velocity ratio.

simple stable kind, the measurements were made up to a Reynolds number of 410. These observations on the nature of the flow near the wall are the main feature of the experimental work and are used in the explanation of transfer processes in a later section.

Some further experiments were made injecting dye at points around the surface. These tests were intended to provide some observations in the wake region far removed from the surface. The rotational motion can be clearly seen from Figs. 7(a), (b), (c) and (d). Superposition of these photographs as shown in Fig. 7(e) show the location of the eye of the vortex. This point is given approximately by the points of intersection of the dye traces as illustrated in the sketch. The dye filaments in these experiments were produced in the manner described under (ii) in "Experimental Apparatus and Procedure". These results must only be used qualitatively, because some unavoidable disturbance of the flow close to the surface, the disturbances produced are short-lived and the results may be used quantitatively.

INTERPRETATION OF THE FLOW NEAR THE SURFACE

In this section, we attempt to account for the distribution and magnitude of the surface transfer rates in the wake. For this purpose we envisage a simple flow model based on our observations. In the experiments to study the nature of the flow close to the surface, a very short filament of dye was carefully injected along the axis of the sphere and its behaviour was examined. At a particular instant the dye line was as shown in Fig. 4, which has been described previously. The sketch, Fig. 8 indicates our impressions of what happens. Maximum velocities appear to occur at a short distance from the surface as in boundary-layer flows. The shape of the dye filament at any instant must be interpreted very carefully as it does not depend on tangential velocities alone. Within the region defined by the line XY and the surface, radial components are probably very small and the dye filament there can be used to measure the average circumferential component of velocity directly. Outside XY, however, there are significant inward radial components with the result as shown. The histories of these typical fluid elements initially

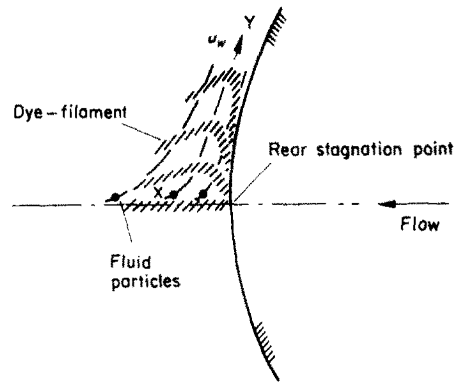


FIG. 8. The behaviour of a dye filament injected into the wake.

located near the axis are probably as illustrated. The circumferential component of velocity does not vary to the same extent outside XY as it does within XY which is typical of a boundary-layer flow. It would appear, therefore, that there is adequate justification for employing a boundary-layer type model for the wake under the particular conditions studied.

To test this idea, we consider first the results for the transfer processes in the upstream boundary-layer region and, in particular, the upstream stagnation point. Concurrent with the present study, this region was considered. To determine the mass-transfer coefficient analytically, von Kármán's integral method for axisymmetric boundary-layer flow was used. In the analysis, fourth order polynominals were employed for the velocity and mass concentration distributions, Tomotika's [6] velocity boundary-layer thickness being used in the evaluation of the mass-transfer boundary layer. Knowing the velocity–mass-transfer boundary-layer thickness ratio the mass-transfer coefficient h_d and hence the Sherwood number Sh_0 was evaluated. For the mass transfer of naphthalene to air for which there exists reliable experimental data, it was found that

$$Sh_0 = 1{\cdot}876\ Re^{0\cdot5} \quad (1)$$

where $$Re = u_\infty d/\nu.$$

It should be observed that the velocity at the edge of the boundary layer, u_1, is related to the main stream velocity, u_∞ by

$$u_1 = u_\infty\ 1{\cdot}5 \sin\theta \quad (2)$$

and is involved in the derivation of equation (1). The average value of u_1, viz. $\bar{u}_1$, is approximately equal to u_∞. This can be shown using equation (2) and

$$\bar{u}_1 = \frac{1}{\theta_s}\int_0^{\theta_s} u_1\, d\theta. \quad (3)$$

It is to be anticipated therefore, that with the concept of a wake boundary layer, the rear stagnation point transfer number $Sh_{0,\,w}$ will be correlated by

$$Sh_{0,\,w} = 1{\cdot}876\ Re_w^{0\cdot5} \quad (4)$$

where $Re_w = \bar{u}_w d/\nu$, with $\bar{u}_w$ the average velocity at the edge of the wake boundary layer. From Fig. 6 it will be seen that the ratio $(\bar{u}_w/u_\infty)$ is approximately constant over the range of Reynolds number studied and has a value of about 0·077. Therefore,

$$Sh_{0,\,w} = 0{\cdot}52\ Re^{0\cdot5}. \quad (5)$$

Equations (1) and (5) are compared with the experimental data of Frössling [7] for mass transfer at the front and rear stagnation points respectively in Fig. 9. The agreement is good for the front stagnation point: for the rear stagnation point the result is encouraging particularly in view of the simplicity of the idea involved in the derivation of equation (4).

The distribution of the transfer number around the surface in the wake region must, with the proposed boundary-layer concept, have the same trend as over the front region. The transfer coefficient decreases towards the separation point as will be seen in Frössling's [7] curves. There is no equivalent simple analysis for determining the local transfer rate, but there is no great disadvantage here because an assumption of a linear decrease from the stagnation value to that at separation would suffice.

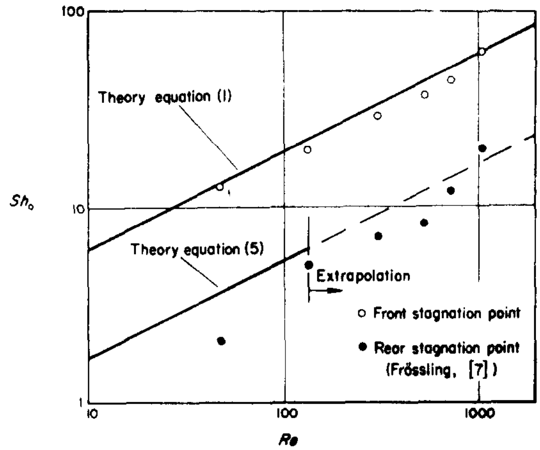


FIG. 9. Mass transfer at the stagnation points of a sphere. A comparison between theory and experiment.

Referring to Fig. 9, it might be that the present idea can be used over a wider Reynolds number range than was first anticipated. There is satisfactory correlation up to Reynolds number equal to 1000 which increases the range of applicability of this idea.

The pattern of the flow in the wake at very large Reynolds numbers, is so complex that it is difficult to suggest a flow model which will facilitate an analysis. It is, however, well known that the transfer rates increase rapidly with the rear stagnation value exceeding that at the front. Because of the wide range of wake transfer rates, therefore the present technique of predicting those at low Reynolds numbers would appear to be a reasonable one.

DISCUSSION AND CONCLUSION

The complicated flow patterns of the fluid in the separated region behind the sphere are not easily handled by conventional analytical methods. As a contribution to the solution of the problem of the sphere wake, experiments have been conducted in the lower velocity range when the flow pattern can be carefully observed. By concentrating attention on the surface effects, the idea of a boundary-layer type flow is introduced which enables a simple analysis to be devised. The theory can only be considered approximate, but it shows satisfactory agreement with experimental data.

The usefulness of the present findings are restricted only in so far as that they have limited practical applications. The results do, however, suggest that in considering transfer processes in other wake situations and higher velocities it is the surface effects which should be studied.

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Résumé—Cet article a trait à une étude du mécanisme de transport dans la région du sillage d'une sphère dans un écoulement à faible vitesse. Les caractéristiques principales de l'étude sont une interprétation de quelques observations visuelles du champ d'écoulement dans le sillage d'une sphère et un examen quantitatif simple des vitesses de transport de chaleur et de masse qui se produiraiet avec le régime particulier d'écoulement. Dans l'étude expérimentale, on a porté l'attention sur l'écoulement immédiatement au voisinage de la surface. Comme résultat de cette technique, un modèle simple pour l'écoulement dans le sillage près de la surface a été imaginé et on a montré qu'il était possible d'avoir avec ce modèle une corrélation satisfaisante des vitesses de transport mesurées dans des études précédentes. Le fluide de travail dans l'étude expérimentale était de l'eau et la valeur maximale du nombre de Reynolds était 410.

Zusammenfassung—Diese Arbeit befasst sich mit der Untersuchung des Übergangsmechanismuses in der Totwasserzone einer Kugel bei niedrigen Strömungsgeschwindigkeiten. Die Hauptkennzeichen der Arbeit sind eine Deutung einiger visueller Beobachtungen des Geschwindigkeitsfeldes im Kielwasser einer Kugel und eine einfache quantitative Untersuchung der Wärme- und Stoffübergangswerte, die bei dem besonderen Strömungssystem auftreten würden. Bei der experimentellen Arbeit wurde die Aufmerksamkeit auf den Flüssigkeitsstrom gelenkt, der unmittelbar mit der Oberfläche in Berührung steht. Als ein Ergebnis dieser Technik wurde ein einfaches Modell für den Totwasserstrom nahe der Oberfläche erstellt. Mit diesem Modell wird gezeigt, dass eine zufriedenstellende Korrelation der experimentell gemessenen Übergangswerte früherer Arbeiten möglich ist. Das Arbeitsmedium bei der experimentellen Untersuchung war Wasser und die Maximalwerte der Reynoldszahlen beliefen sich auf 410.

Аннотация—Эта статья посвящена изучению механизмов переноса в кильватере шара при течении с низкой скоростью. Основная задача работы дать интерпретацию некоторых визуальных наблюдений за полем скорости в кильватере шара и провести простое количественное исследование скоростей переноса тепла и массы, имеющего место при данном режиме течения. При проведении экспериментов внимание обращалось на течение жидкости непосредственно у поверхности. В результате разработана простая модель течения в кильватере у поверхности и показано, что эта модель дает возможность получить удовлетворительную корреляцию экспериментальных величин скоростей переноса, замеренных в ранее проводимой работе. В экспериментах в качестве рабочей жидкости бралась вода; максимальная величина критерия Рейнольдса равнялась 410.